



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 34

Forward Pricing – Investment Assets

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PAYOFF & PROFIT DIAGRAM

- The price of the underlying invariably evolves as a stochastic process. Hence, its value on the maturity of the derivative is a random variable. The payoff from the derivative is, consequently, the function of a random variable. It is, therefore, useful to ascertain the payoff from the derivative corresponding to different possible values of the underlying's price. This is usually done through payoff diagrams.

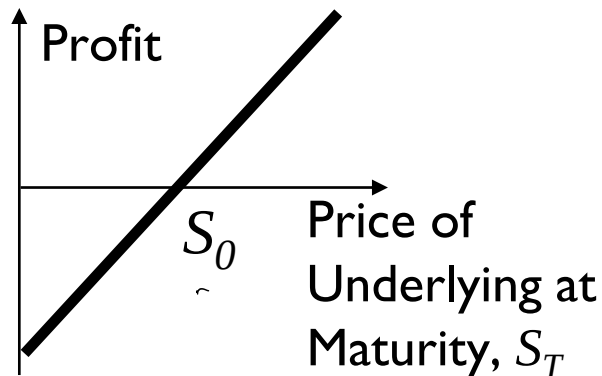
PAYOFF DIAGRAM...

- A payoff (profit) profile is the graphical representation of the payoff (profit) under the contract as a function of the market price of the underlying asset.
- A payoff (profit) function is the mathematical representation of the payoff (profit) under the contract as a function of the market price of the underlying asset.

PAYOFF OF ASSET POSITIONS

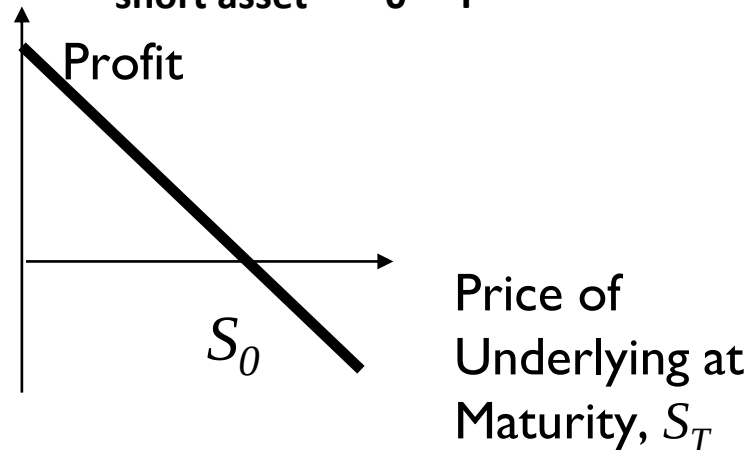
LONG ASSET

- $\Pi_{\text{long asset}} = S_T$
- $\pi_{\text{long asset}} = S_T - S_0$



SHORT ASSET

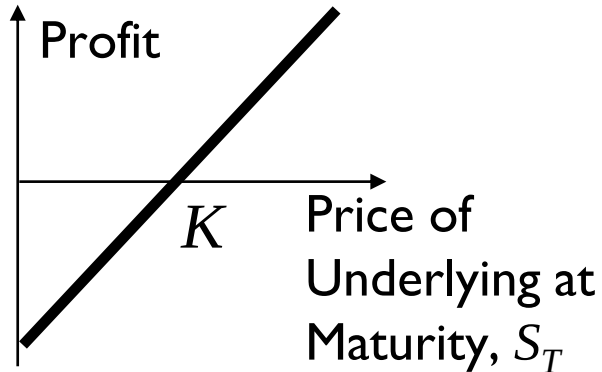
- $\Pi_{\text{short asset}} = -S_T$
- $\pi_{\text{short asset}} = S_0 - S_T$



PROFIT DIAGRAM: LONG & SHORT FORWARD

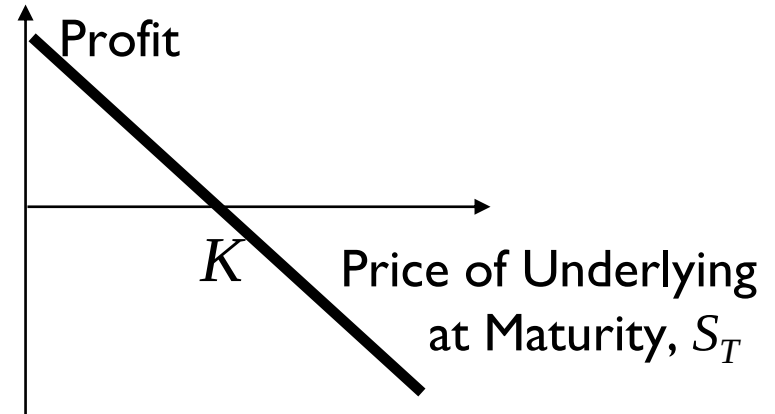
LONG FORWARD

- $\Pi_{\text{long forward}} = S_T - K$
- $\pi_{\text{long forward}} = S_T - K$



SHORT FORWARD

- $\Pi_{\text{short forward}} = K - S_T$
- $\pi_{\text{short asset}} = K - S_T$



FORWARD PRICING: ASSET CLASSIFICATION

- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)



NO ARBITRAGE PRICING OF FORWARDS: INVESTMENT ASSETS

- **We start with the simplest case:**
- **No income from the underlying during life of forward;**
- **No carrying cost of underlying during this period;**
- **No transaction costs & market frictions (bid-ask spread, lending-borrowing spread, commissions etc.).**



ARBITRAGE FREE PRICING

	t=0	t=T
BORROW	$+S_0$	$-S_0 \exp(rT)$
BUY STOCK	$-S_0$	
SHORT FORWARD	0	F_0
TOTAL	0	$F_0 - S_0 \exp(rT)$
$F_0 = S_0 \exp(rT)$		

WHY RISK FREE RATE?

- To understand, why we use riskfree rate, we consider a modified version of arbitrage:
- Take a short forward position at price F_0 with maturity T .
- Borrow S_0 , buy asset, hold for time T and deliver against short forward position.
- We assume that the forward is default free. Thus, S will be delivered and F_0 cash will be received at $t=T$ with certainty. Because, the repayment of S_0 (with interest) is to be made out of F_0 , therefore, this repayment is also assumed default free.

FORWARD PRICING WITH DIVIDENDS

Dividend D_τ recd at $t = \tau \in (0, T)$; $D_0 = PV(D_\tau)$; $S_{\text{eff}} = S_0 - D_0$

- Let us assume that the holding of S generates a dividend D_τ at a time τ in $(0, T)$. Let D_0 be the present value of this dividend i.e. D_0 is the amount that can be borrowed against this projected dividend.
- Then, net amount to be borrowed for acquiring one unit of the asset becomes $(S_0 - D_0)$.
- Thus, amount to be repaid $(S_0 - D_0)\exp(rT)$ because repayment of D_0 with interest will be met by D_τ .
- Thus, no arbitrage eq. becomes $F_0 = (S_0 - D_0)\exp(rT)$

FORWARD PRICING (DOLLAR RETURN)

	$t=0$	$t=\tau$	$t=T$
BORROW	$+S_0$		$-S_0 \exp(rT)$
BUY STOCK	$-S_0$		
RECEIVE DIVIDEND		D_τ	
INVEST DIVIDEND		$-D_\tau$	$+D_T$
SHORT FORWARD	0	0	F_0
TOTAL	0	0	$F_0 + D_T - S_0 \exp(rT)$
$F_0 = S_0 \exp(rT) - D_T = (S_0 - D_0) \exp(rT)$			



FORWARD PRICING WITH YIELD

$$\exp(-qT) <- 1; \text{Cost} = S_0 \exp(-qT); F_0 = [S_0 \exp(-qT)] \exp(rT)$$

- Let us assume that the holding of S generates a yield q per unit time. Thus, if we want to have one unit of at time T for delivery against the forward contract, we need to have only $\exp(-qT)$ units now.
- Then, amount to be borrowed for acquiring $\exp(-qT)$ of the asset spot becomes $S_0 \exp(-qT)$.
- Thus, amount to be repaid $S_0 \exp[(r-q)T]$.
- Thus, no arbitrage eq. becomes $F_0 = S_0 \exp[(r-q)T]$

FORWARD PRICING (CONTINUOUS COMPOUNDED YIELD)

	$t=0$	$t=T$
BORROW	$+S_0$	$-S_0 \exp(rT)$
BUY STOCK	$-S_0$	
SHORT FORWARD ON $\exp(qT)$ UNITS	0	$F_0 \exp(qT)$
TOTAL	0	$F_0 \exp(qT) - S_0 \exp(rT)$
$F_0 = S_0 \exp[(r-q)T]$		



PRICING WITH CARRYING COSTS

- $F_0 = (S_0 + U_0) \exp(rT)$
- $F_0 = S_0 \exp[(r + q)T]$



CASH & CARRY ARBITRAGE

- Transaction costs, costs of carrying the asset are ignored to keep the exposition tractable.
- F_0 is the arbitrage free forward price.
- F^*_0 is the actual forward price.
- Let $F^*_0 > F_0 = S_0 \exp(rT)$.



- Consider the following strategy:
- (t=0)
- Borrow S_0
- Buy asset and hold the asset for (0,T)
- Take a short forward position.
- (t=T)
- Deliver the asset against short forward position,
- Receive cash F^*_0 ,
- Pay off loan with interest $S_0 \exp(rT)$.
- Since $F^*_0 > F_0 = S_0 \exp(rT)$ you end up with positive cash.

REVERSE CASH & CARRY ARBITRAGE

- Let $F^*_0 < F_0 = S_0 \exp(rT)$.
- Consider the following strategy:
- (t=0) Borrow the asset S and sell for S_0 , invest this amount at r% for (0,T). Take a long forward position.
- (t=T) Receive invested amount with interest $S_0 \exp(rT)$. Pay F^*_0 against forward and receive the asset, restore it to owner.
- Since $F^*_0 < S_0 \exp(rT)$ you end up with positive cash.